# ADDENDUM

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The subject matter of the Addendum, as already stated in the text of the research paper, is certain fundamental concepts like theory of psychological measurement, distribution of scores, averages or means, variance, standardized scores, the correlation coefficient, mathematical model for factor analysis, the correlation matrix, the method of principal components in factor analysis and the rotation of factors, which are considered as 'elements' of the method of analysis employed in the paper. They are included as part of the paper so as to facilitate ready reference for some of our musicologists who are not used to the type of 'vocabulary' used in our research.

**On Measurement:** Measurement is the assignment of numerals to objects or events, according to rules. Numerals are merely symbols, scratches on paper, each being a distinct form not identical with any other. Such assignment has two purposes. First, when numerals have been assigned to represent quantities of objects or things, psychological or otherwise, manipulations of such quantities may be carried out symbolically rather than physically. Secondly, with the vast amount of knowledge already gained by mathematicians on the number systems new relationships and insight can be derived by operating, under the rules of mathematics, with those numerals assigned to psychological objects or things. After all, scientists have become fully aware that mathematics, with its number system is a game of signs and rules, man-made and arbitrary, like the game of chess (Guilford, 1954; Siegel, 1956).

However, before the number system may be employed to describe quantities of some particular variables, it is necessary to demonstrate that the variables possess the kind of intrinsic structure which the number system itself has. That is, there must be a similarity of structural relations between the mathematical system of number and the physical

system of quantity. In practice it is never necessary to go to this extreme and, as a matter of fact, it would be impossible to do so, for there are some postulates underlying the system of rational numbers which could not be verified completely in any empirical context (Guilford, op. cit.).

Reverting back to measurement, since there are different kinds of rules, there are different kinds of measurements; but for each kind some degree of isomorphism obtains between the empirical relation among objects and the formal relations among numbers. Among the empirical properties of the world for which numbers may serve as models the most important are identity, order, intervals and ratios. Corresponding to each of these uses there is a type of scale of measurement, namely nominal, ordinal, interval and ratio, whose classification with examples is given below. (Garrett, 1958; Guilford, op. cit.; Loevinger, 1951; Seigel, op. cit.; Sprott, 1964; Wood, 1960).

| | | |
|---|---|---|
| identity (nominal) | : | numbers may serve as names or labels to identify item or classes; "numbering" of football players. |
| order (ordinal) | : | numbers may serve to reflect the rank order of items; "1" st class, "2"nd class performance. |
| interval | : | numbers may serve to reflect the differences or distances among items; Fahrenheit or centigrade temperature scale, calendar time — "1967", "1889". In this case arbitrary zero has been set by convention only, because as will be noticed "0" (zero) A.D. does not mean 'nothingness' of existence of life or history. |
| ratio | : | numbers may serve to reflect the ratios among items; all physical measurements like "60" inches of height, "100" kg. of weight etc. Here zero is absolute zero, that is, "0" height means "nothingness" of height. In this case it is possible to equate meaningfully ratios of numbers on the scale, e.g., the ratio 12:8 is equal to the ratio 3:2 and both |

stand for the same relation between two real quantities. Such is not the case for the earlier three scales.

The purpose of the above brief description is to state the underlying assumptions in psychological measurements; it is customary to do so in case of propositions for which the evidence is a mathematical proof, because such statements strengthen the consideration of their applicability in context. Thus it becoms clear that measurement may be of several kinds and may be taken to various degrees of precision. The measurement of the power of music to arouse feelings and emotions in the listener by means of adjective scales described above and expressed as a score constitute more or less an interval scale. These scores may be added or subtracted just as we add or subtract inches (ratio scale) but we cannot say that a score of 6 is twice as good as a score of 3, as neither is taken from a zero of just no 'aroused feeling' on the adjective scale. Further, in terms of interval scale, it is justified to use statistical procedures like mean, variance and product moment, correlation coefficient and other statistics that depend on these values which form the basic data of factor analysis. The glossary of these terms now follows.

**Glossary of Basic Statistics for Factor Analysis**

*Distribution of Scores:* Data collected from the experiment in terms of scores of $N$ (herein, 37) individuals on $n$ (herein, 22) adjective scales is known as the distribution of scores. A convenient algebraic notation of such a distribution may be:

| *1st Scale* | *2nd Scale* | .. | *nth Scale* |
|---|---|---|---|
| $X_{11}$ | $X_{21}$ | .. | $X_{n1}$ |
| $X_{12}$ | $X_{22}$ | .. | $X_{n2}$ |
| .. | .. | .. | .. |
| .. | .. | .. | .. |
| $X_{1N}$ | $X_{2N}$ | .. | $X_{nN}$ |

Such a distribution when arranged and classified in some systematic way, like score of 1 by two individuals, score of 2 by seven individuals

score of 3 by 14 individuals and so on, in terms of the 'frequency' of occurrence of scores is referred to as the frequency distribution of scores. The arrangement of raw distribution into frequency distribution is the first step statisticians take in systematizing data in an effort to analyse and interpret the results of an experiment.

*Mean:* Popularly known as average, it is a representative of the values of all the scores made by the group. The statistical process of extracting information from resulting data includes this step of 'reduction of data'. Averages reduce the information contained in $N$ scores into a single measure — measure of central tendency — around which most of the values of the scores lie and as such gives a concise description of the performance of the group as a whole. Such a measure enables us to compare two or more groups on one indicator or same group on two or more indicators. Without going into the types of such measures and their pros and cons, the mathematical notation of the arithmetic mean used in the study may be given as,

$$M_j = \frac{X_{j1} + X_{j2} + X_{j3} + X_{j4} + \dots + X_{jN}}{N}$$

where $M_j$ represents the mean of scores on the $j$ th adjective scale

In summation notation ($\Sigma$) which means summing over all values of $N$, the above formula is written as,

$$M_j = \frac{\Sigma X_{ji}}{N} \qquad (i = 1, 2, \dots\dots N)$$

*Variance:* Next step in analysing the data is to find some measure of the 'variability' of our scores, i.e., of the 'scatter' or 'deviation' of the separate scores around their central tendency. Comparison of two distributions of scores through their means is only partial. In a case of equal averages (say, 4) of two distributions, so far as the means go there may appear to be no difference in the feelings of two groups but on closer inspection of data, it may appear that in the first group the scores ranged from 3 to 5 while in the second group they ranged from 1 to 7. This difference in range shows that the second group "covers more

territory" and is more 'variable' or heterogeneous in scoring. The first group is referred to as more homogeneous or its variability in scores is small. Range is a crude measure of variability in a distribution. A sophisticated and stable measure evolved and extensively used by statisticians is known as variance and is defined as the averaged sum of squares of deviations of scores from their mean. Using the conventional symbol of Greek letter sigma ($\sigma$), its mathematical notation will be,

$$\text{Variance} = \sigma^2_j = \frac{\Sigma\,(X_{ji} - M_j)^2}{N}$$

*Standardized Scores:* Referred to as $Z$-scores, standardized scores are obtained from raw scores through a simple mathematical transformation shown below:

$$Z_{ji} = \frac{X_{ji} - M_j}{\sigma_j}$$

This transmutation does not change the shape of the distribution. The uses of such a shift results in a little convenience in interpreting/comparing results; all distributions of $Z$-scores have a zero mean and unit variance.

*Correlation Coefficient:* The relationship between two sets of scores on various adjective scales are ascertained statistically in the form of an index referred to as correlation coefficient and denoted by the the letter $r$. The formula for this measure is,

$$r_{jk} = \frac{\Sigma\, Z_{ji}\, Z_{ki}}{N}$$ (where $j$ and $k$ represent the two sets of scores)

In relation to the above described measures of central tendency and variability, $r$ may be defined as the averaged sum of the products of $z$-scores on two scales between which the relationship is being measured. Such a value varies from $-1$ to $+1$ according as the relationship is negatively perfect (high score on one scale, low on the other for a particular individual) to positively perfect (high score on one scale, high on

the other for a particular individual), no relationship being indexed by the number zero. Between these two limits increasing degree of positive relationship may become indicated by such coefficients as 0·33, 0·65, 0·92, and increasing degrees of negative relationships may become indicated by such coefficients as —0·18, — 0·47, —0·89.

Such correlation coefficients representing the measurement of intercorrelations among all the adjective scale score distributions of a study constitute the basic data for factor analysis. Because, as long as such coefficients have values sufficiently more or less than zero (case of no relationship), it leads one to believe that the two activities have in common some fundamental functions referred to erlier by us as the core of the concepts. Once the correlations are determined, the scores of individuals usually are not referred to again.

**Mathematical Model for Factor Analysis**

It is the object of a factor analysis to represent an indicator $Z_j$ in terms of several underlying 'factors'. The simplest mathematical model for describing a variable in terms of several others is a linear one and that is the fundamental assumption underlying all of the present-day factor analysis methods. The terms of this linear model mean that an individual's score on an indicator is the weighted sum of his scores on the separate components which enter into performance on that indicator. A given component or factor may enter into the scores of several indicators; in fact, its nature can be explored only if it does. Algebraically, we may write the basic assumption as follows:

$$Z_{ji} = a_{ji} F_{1i} + a_{j2} F_{2i} + \dots\dots\dots\dots a_{jm} F_{mi} + U_j.$$

In this equation $Z_{ji}$ refers to the standard score of the individual $i$ on the indicator $j$. The quantity $F_{1i}$ refers to the score of the individual on the first factor, this score also being expressed in standard form. The coefficient $a_{j1}$ refers to the "weight" or "loading" of the first factor in the indicator $j$; the larger the coefficient $a_{jf}$ the more important is factor $f$ in determining the total score on indicator $j$. There are $m$ common factors. $U_j$ is known as the unique factor in test $j$ which means that it is involved only in one indicator namely, $j$. The factor score is a

characteristic of the individual and will have the same numerical value for the individual in all indicators in which that factor occurs. The coefficients of different factors characterize the indicator and are the same for all individuals.

The immediate purpose of factor analysis is to assign numerical values to the coefficients, the *a*'s, in this equation. In order to accomplish this purpose one must have a sample of individuals who all attempt several indicators inovlving more or less the same factors. The relationships between the scores on the various indicators are then ascertained in terms of correlation coefficients. These intercorrelations are presented in a tabular form as:

**Correlation Matrix**

| *Indicators* | 1 | 2 | 3 | 4 | .. | .. | *n* |
|---|---|---|---|---|---|---|---|
| 1 | | $r_{12}$ | $r_{13}$ | $r_{14}$ | .. | .. | $r_{1n}$ |
| 2 | $r_{12}$ | | $r_{23}$ | $r_{24}$ | .. | .. | $r_{2n}$ |
| 3 | $r_{13}$ | $r_{23}$ | | $r_{34}$ | .. | .. | $r_{3n}$ |
| 4 | $r_{14}$ | $r_{24}$ | $r_{34}$ | | .. | .. | $r_{4n}$ |
| .. | .. | .. | .. | .. | | .. | .. |
| *n* | $r_{1n}$ | .. | .. | .. | | .. | $r_{nn}$ |

Since there is only one correlation between any given pair of indicators, correlation matrix is 'symmetrical about the principal diagonal'. The analysis cannot be carried out until some quantities are entered in the principal diagonal. Different entries were suggested by theoreticians on different grounds, though, on the face of it, unity should be entered in the diagonal since the correlation of a set of scores with itself should be one. These and some other considerations generated heated and inspired controversies about the "best" method of factor analysis. However, by this time it has become evident that different types of factorial solutions correspond to the different mathematical theories in the description of a particular scientific problem. A brief introduction to the method of 'Principal Components' will now be given.

**Method of Principal Components**

If one were to make his choice entirely upon statistical considerations, a rather natural approach would be to represent the original set of variables in terms of a number of factors, determined in sequence so that at each successive stage the factor would account for a maximum of the variance. This statistically optimal solution was first proposed by Pearson at the turn of the century, and in the 1930's Hotelling provided the full development of the method. While this procedure is very straightforward, it entails a very considerable amount of computations and becomes impractical with ordinary computing facilities when the matrix is of order 10 of greater. This difficulty has however been overcome by the use of high-speed electronic computer.

The reference system of the principal factor solution is the criterion of the 'ellipsoidal fit'. When the factors are represented by the principal axes of the ellipsoids, each successive one contributes a decreasing amount to the total communality. In other words, the first principal factor accounts for the maximum possible variance; the second factor accounts for a maximum in the residual space with the first factor removed; the third factor, a maximum in the residual space excluding the first two factors; and so on until the last common factor accounts for whatever remains. The factors are determined in such a way as to be uncorrelated. There are as many factors as there are indicators, but the last few are likely to be of small importance. Unique facotrs are assumed to be non-existent or of just the magnitude that would be accounted for by the unreliability of the various test indicators.

The method of principal factor solution can account for both negative as well as positive correlation coefficients. Accordingly, approximately half of the 'loadings' of each of the factors are negative that is, *F*'s may be bipolar factors. A bipolor factor is not essentialy different from any other but is merely one for which several of the variables have significant negative projections. Such variables may be regarded as measuring the negative aspect of the usual type of factors. Thus, if a number of variables identified with "fear" are represented by positive projections, variables with negative projections might be interpreted as measuring "courage".

The above description has given the general appearance and properties of a principal factor solution. Its mathematical derivation and computing procedures are too complex to be presented here. Programmes for Electronic computors are available for solving the matrix. Flow chart for programming the principal factor solution is also provided by Harman wherein the mathematical derivation is explained.

*En passant,* enumeration of chief among the other procedures for extracting factors may be mentioned. These include, Spearman's Two-factor solution, Holzinger's Bi-factor solution, and Thurston's Centroid method.

*Rotation of Factors:* The end product of these solutions, generally are not acceptable to psychologists, although, some prefer the principal factor solution. In search of "meaningful" factors, psychologists have introduced various theories in the hope of arriving at a form of solution which would be unique and apply equally well to any branch of human knowledge other than their own. Within the scope of these theories methods of transforming some initial factor solution to the another 'preferred' type of solution were tried, the sole purpose being increasing the meaningfulness of the leadings. Such transformations were given the name of 'rotation' making use of geometric representation of factor analysis.

Some graphical methods were tried earlier but have now been discarded because they used subjective judgments. Many new 'analytic' methods have been evolved. 'Varimax criterion of Rotation' is one of them. Given by Kaiser, it defines the simplicity of a factor as the variance of its squared leadings and argues that when this variance is at a maximum the factor has the greatest simplicity in the sense that its components (rotated loadings) tend toward unity and zero. Hence the name, Varimax. It may be noted that this method is adapted to the assumption of orthogonal simple structure.

*Conclusion:* Further explanation as also the statistical test of hypotheses in factor analysis are not included within the scope of the present addendum. For fuller details Harman is the standard text. (Harman, op. cit.).

## Addendum References

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